1) http://www.artofproblemsolving.com/community/c6h161059p1205

Let a, b, c be positive real numbers. Prove the inequality
$$\frac{1}{a\left(b+1\right)}+\frac{1}{b\left(c+1\right)}+\frac{1}{c\left(a+1\right)}\geq\frac{3}{1+abc}.$$

2) Для додатних чисел $x_1, x_2 \dots, x_n$ доведіть нерівність

$$(1+x_1)(1+x_1+x_2)\dots(1+x_1+x_2+\dots+x_n)\geq\sqrt{(n+1)^{n+1}x_1x_2\dots x_n}$$

3) http://www.artofproblemsolving.com/community/c6h60434p365178

Let a, b, c be positive real numbers such that $abc=1$. Prove that
$$\frac{1}{a^{3}\left(b+c\right)}+\frac{1}{b^{3}\left(c+a\right)}+\frac{1}{c^{3}\left(a+b\right)}\geq\frac{3}{2}.$$

4) Положительные числа *a*, *b*, *c*, *x*, *y*, *z* удовлетворяют условию $a+b+c = x+y+z$. Докажите, что $ax(a+x)+by(b+y)+cz(c+z) \geq 3(abc+xyz)$.

5) http://www.artofproblemsolving.com/community/c6h355781p1932917

Let a, b, c be positive real numbers such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=a+b+c$. Prove that:
$$\frac{1}{(2a+b+c)^2}+\frac{1}{(a+2b+c)^2}+\frac{1}{(a+b+2c)^2}\leq\frac{3}{16}$$

6) http://www.artofproblemsolving.com/community/c6h287855p1555899

Let a, b, c, d be positive real numbers such that $abcd=1$ and $a+b+c+d>\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}$. Prove that
$$a+b+c+d<\frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{a}{d}$$

7) http://www.artofproblemsolving.com/community/c6h418680p2362280

Let $x_1,\ldots,x_{100}$ be nonnegative real numbers such that $x_i+x_{i+1}+x_{i+2}\leq 1$ for all $i=1,\ldots,100$ (we put $x_{101}=x_1, x_{102}=x_2$). Find the maximal possible value of the sum $S=\sum_{i=1}^{100}x_ix_{i+2}$.

8) http://www.artofproblemsolving.com/community/c6h597119p3543341

Prove that in any set of 2000 distinct real numbers there exist two pairs $a>b$ and $c>d$ with $a\neq c$ or $b\neq d$, such that $\left|\frac{a-b}{c-d}-1\right|<\frac{1}{100000}$.

9) http://www.artofproblemsolving.com/community/c6h215222p1190551

(i) If x, y and z are three real numbers, all different from 1, such that $xyz = 1$, then prove that

$$\frac{x^2}{(x-1)^2} + \frac{y^2}{(y-1)^2} + \frac{z^2}{(z-1)^2} \geq 1.$$

(With the $\sum$ sign for cyclic summation, this inequality could be rewritten as $\sum \frac{x^2}{(x-1)^2} \geq 1$.)

(ii) Prove that equality is achieved for infinitely many triples of rational numbers x, y and z.

10) http://www.artofproblemsolving.com/community/c6h418679p2362276

Let the real numbers a, b, c, d satisfy the relations $a+b+c+d=6$ and $a^2+b^2+c^2+d^2=12.$ Prove that

$$36 \leq 4\left(a^3+b^3+c^3+d^3\right) - \left(a^4+b^4+c^4+d^4\right) \leq 48.$$

11) http://www.artofproblemsolving.com/community/c6h355782p1932920

Let a, b, c be positive real numbers such that $ab+bc+ca \leq 3abc$. Prove that

$$\sqrt{\frac{a^2+b^2}{a+b}}+\sqrt{\frac{b^2+c^2}{b+c}}+\sqrt{\frac{c^2+a^2}{c+a}}+3 \leq \sqrt{2}\left(\sqrt{a+b}+\sqrt{b+c}+\sqrt{c+a}\right)$$

12) http://www.artofproblemsolving.com/community/c6h289590p1566061

Prove that for positive real numbers x, y, z,

$$x^3(y^2+z^2)^2+y^3(z^2+x^2)^2+z^3(x^2+y^2)^2 \geq xyz[xy(x+y)^2+yz(y+z)^2+zx(z+x)^2].$$